How do magnetic microwires interact magnetostatically?

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The magnetostatic interaction between two ferromagnetic microwires is calculated as a function of their geometric parameters and compared with those measured through magnetic hysteresis loops of glass-coated amorphous $Fe_{77.5}Si_{7.5}B_{15}$ microwires. The hysteresis loops are characterized by well-defined Barkhausen jumps corresponding each to the magnetization reversal of individual microwires, separated by horizontal plateaux. It is shown that the magnetostatic interaction between them is responsible for the appearance of these plateaux. Finally, using the expression for the magnetostatic interaction is trivial to obtain the interacting force between microwires. Our results are intended to provide guidelines for the use of these microwires with technological purpose such as the fabrication of magnetic sensors. © *2009 American Institute of Physics*.
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I. INTRODUCTION

In the past decades, soft magnetic materials have been deeply investigated. Besides the basis scientific interest in their magnetic properties, there are a great deal of technologic interest due to their use in sensing applications, particularly in the fields of automotive, mobile communication, medical, and home appliance industries.[1–5] Moreover, these materials are very promising for spintronic devices in magnetic recording media.[6] Two types of soft magnetic microwire families are currently studied: in-water quenched amorphous wires with diameters of around 120 μm,[7] and quenched and drawn microwires with diameters ranging from around 2 to 20 μm,[8] covered by a protective insulating glassy coat. Bistable microwires are characterized by square-shaped hysteresis loops defined by the abrupt reversal of the magnetization between two stable remanent states.[9]

Although an array composed of a few ferromagnetic wires could in principle seem a quite simple problem to study and model, it is striking to notice how complex this problem can turn out to be. The effects of interparticle interactions are in general complicated by the fact that the dipolar fields depend on the magnetization state of each element, which in turn depends on the fields due to adjacent elements. Therefore, the modeling of interacting arrays of wires is often subject to strong simplifications, for example, modeling the wire using a one-dimensional modified classical Ising model.[10,11] Zhan *et al.*[12] used the dipole approximation including additionally a length correction. Velazquez and Vazquez[13,14] considered each microwire as a dipole, in a way that the axial field generated by a microwire is proportional to its magnetization. Nevertheless, this model is merely phenomenological since the comparison of experimental results with a strictly dipolar model shows that the interaction in the actual case is more intense. They also calculated the magnetostatic field and expanded it in multipolar terms,[15] showing that the nondipolar contributions of the field are non-negligible for distances considered in experiments. Besides, the magnetostatic interaction energy between two magnetic elements of arbitrary shape was derived within the framework of a Fourier space approach by De Graef and co-workers.[16,17] Recently, a detailed study of the magnetostatic interaction between two parallel wires placed side by side has been shown.[18,19] In spite of the extended study of the dipolar interactions, the effect of magnetostatic interactions on the hysteresis loops, due to the vertical displacement of the wires, has not been studied yet.

The purpose of this work is to develop an analytical model for the full long-range magnetostatic interaction between two microwires exploring the possibility of varying the magnetic coupling as a function of the wires' position. The geometry of the wires is characterized by their external radii R and length L, as depicted in Fig. 1. The separation

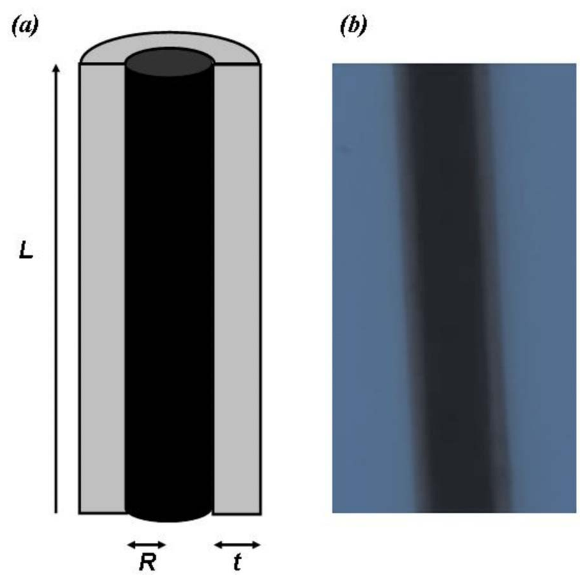

FIG. 1. (Color online) (a) Geometric parameters used for the individual wire description. (b) Micrograph image that shows a glass-coated amorphous microwire.

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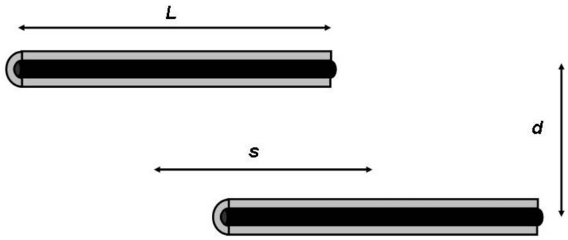

FIG. 2. Relative position of interacting wires: d is the interaxial distance and s is the horizontal separation.

between the wires is written in terms of the interaxial distance, d, and the horizontal separation, s, as depicted in Fig. 2. Our model goes beyond the dipole-dipole approximation and leads us to obtain an analytical expression for the interaction in which the lengths and radii of the wires are taken into account. We focus on the interacting force in pairs of interacting wires, as a function of the distance between them, in order to gain insight on the understanding of the role of interactions on the plateaux that appears in the hysteresis loops.

II. EXPERIMENTAL METHODS

The experimental measurements have been performed in glass-coated amorphous bistable magnetic microwires with nominal composition $Fe_{77.5}Si_{7.5}B_{15}$, with a saturation magnetization $M_0=1.2\times10^6$ A/m, radii of $R=6.5$ μm, and the thickness of the glass coating of $t=3.3$ μm. They are fabricated by means of Taylor–Ulitovsky technique by which the molten metallic alloy and its glassy coating are rapidly quenched and drawn to a kind of composite microwire. Two samples with length of $L=7$ mm each were cut from a larger wire and fixed parallel by means of vacuum grease on a glass holder, with a separation of $d=45$ μm between the wires.

The morphology of microwires was investigated by a retro-optical microscope (Olympus). The hysteresis curves were measured on a specially designed vibrating sample magnetometer, inserted within a pair of Helmholtz coils. These coils have sufficient field to saturate the wires and present the advantage of field homogeneity and the absence of remanent fields. We will focus our attention on measurements performed at room temperature because a low temperature there is a change in the domain structure of the microwires, probably owing to the increasing internal stresses induced by the different thermal expansion coefficients of the ferromagnetic alloy and the covering glass.[10]

III. THEORETICAL CALCULATIONS

We adopt a simplified approach in which the discrete distribution of magnetic moments of microwire is replaced with a continuous one characterized by a slowly varying magnetization $\mathbf{M}(\mathbf{r})$. We consider wires for which $L \gg R$ so it is reasonable to assume an axial magnetization due to shape anisotropy, defined by $\mathbf{M}(\mathbf{r})=M_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector parallel to the wire axis. The magnetostatic interaction between wires can be calculated from[20]

$$E_{\text{int}} = \mu_0 \int \mathbf{M}_2(\mathbf{r})\, \nabla\, U_1(\mathbf{r}) dV,$$

where $\mathbf{M}_2(\mathbf{r})$ is the magnetization of wire 2 and $U_1(\mathbf{r})$ is the magnetostatic potential of microwire 1. The expression for this potential is given by

$$U = \frac{1}{4\pi}\left(-\int_V \frac{\nabla\cdot\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r' + \int_S \frac{\hat{\mathbf{n}}'\cdot\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}ds'\right). \quad (1)$$

Note that the first term on the left-hand side of Eq. (1) vanishes, because the magnetization field is constant; furthermore, in the surface integral of Eq. (1), the only contributions arise from the upper and lower bases of the wire: The upper circle is located at $z=L/2$ and the lower circle is located at $z=-L/2$. Due to the symmetry of the problem, we used the adequate type of the cylindrical kernel for the integral,[21] and after few manipulations, one finds that the integral expression for the scalar potential is given by

$$U(r,z) = \frac{M_0 R}{2}\int_0^\infty \frac{dk}{k} J_0(kr) J_1(kR)\left(e^{-k|L/2-z|} - e^{-k|-L/2-z|}\right), \quad (2)$$

where J_m is a Bessel function of first kind and m order.

Now it is possible to calculate the magnetostatic interaction energy between two identical microwires using the magnetostatic field experienced by one of the wires due to the other. The final results read as

$$E_{\text{int}} = -\mu_0 M_0^2 \pi R^2 L \int_0^\infty \frac{dq}{q^2} e^{-q(1+s/L)} J_0\left(q\frac{d}{L}\right) J_1^2\left(q\frac{R}{L}\right) \times \begin{cases} (1-e^q)^2, & s \geq L \\ (1-2e^q+e^{2q(s/L)}), & s \leq L. \end{cases} \quad (3)$$

Equation (3) has been previously obtained for nanotubes.[22] The general expression for the interaction energy between wires with axial magnetization, given by Eq. (3), can only be solved numerically. However, wires that motivated this work satisfy $R/L=\alpha \ll 1$, in which case one can use that $J_1(\alpha x) \approx \alpha x/2$. With this approximation, Eq. (3) can be written in a very simple form as

$$E_{\text{int}} = -\mu_0 M_0^2 \frac{\pi R^4}{4}\left(\frac{1}{\sqrt{d^2+(L-s)^2}} - \frac{2}{\sqrt{d^2+s^2}} + \frac{1}{\sqrt{d^2+(L+s)^2}}\right). \quad (4)$$

Finally, the magnetostatic field can be written as a function of the magnetostatic interaction between the magnetic microwires and is given by[5]

$$H_{\text{int}} = \frac{E_{\text{int}}}{\mu_0 M_0 V} = -\frac{M_0 R^2}{4L}\left(\frac{1}{\sqrt{d^2+(L-s)^2}} - \frac{2}{\sqrt{d^2+s^2}} + \frac{1}{\sqrt{d^2+(L+s)^2}}\right). \quad (5)$$

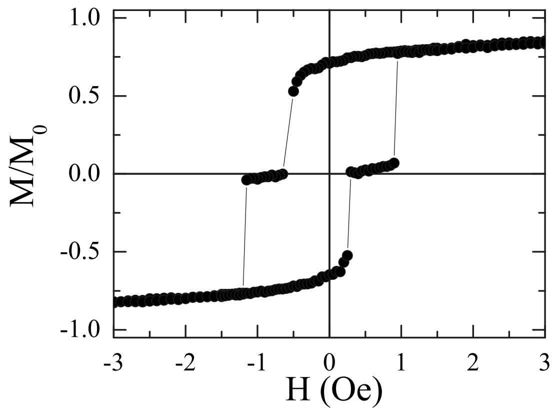

FIG. 3. Hysteresis loop for two parallel amorphous wires of $Fe_{77.5}Si_{7.5}B_{15}$.

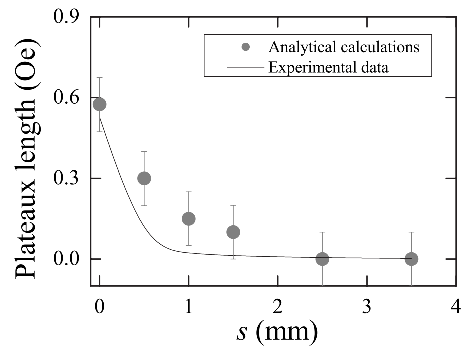

FIG. 5. Magnetostatic field as a function of the horizontal separation of the microwires. The gray dots correspond to experimental data and the solid line represents the values calculated analytically. Parameters: $R=6.5$ μm, $L=7$ mm, $t=3.3$ μm, $d=45$ μm, and s ranging between 0 and 3.5 mm. For the analytical calculations we have used an effective radius of $R^*=4.7$ μm and $M_0=1.2\times 10^6$ A/m.

IV. RESULTS

Figure 3 shows the hysteresis loop for two microwires of radii of $R=6.5$ μm, length of $L=7$ mm, thickness of the glass coating of $t=3.3$ μm, and interaxial separation $d=45$ μm at room temperature. As observed, the magnetization process takes place in two steps: a jump near 0.5 Oe, which must be ascribed to the reversion of a microwire; and an additional jump near 1.2 Oe, corresponding to the reversion of the other microwire. The flat region before the second step corresponds to the magnetic field felt by one wire due to the other.

A. Dependence of the interaxial separation of the wires on the magnetostatic field

Theoretical and experimental magnetostatic results are combined in Fig. 4. Experimental data for the magnetostatic field for different values of the interaxial separation d are depicted by gray dots and the theoretical prediction is represented by the solid line. For the analytical calculations we have used an effective radius of $R^*=4.7$ μm, which is smaller than the real radius, in order to compensate the approximation of considering a homogeneous magnetization in our model.[19] It is clear that there is a strong dependence of the magnetostatic field on d. Note the good agreement between experimental datapoints and analytical results for distances smaller than 80 μm. The coupling rapidly decreases by increasing the separation between the wires, and becomes negligible for interaxial distances bigger than 80 μm, at least within our experimental sensitivity. It can be understood if we consider that the stray field felt by one wire due to the other changes considerably if the wires are not completely parallel to each other. Thus, when the separation between the wires is big enough, small deviations may reduce the stray field produced by the wires.

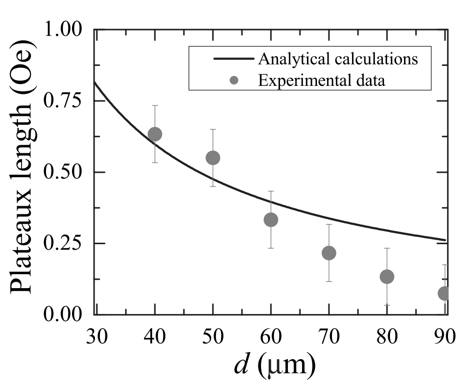

FIG. 4. Magnetostatic field as a function of the interaxial separation of the microwires. The gray dots correspond to experimental data and the solid line represents the values calculated analytically. Parameters: $R=6.5$ μm, $L=7$ mm, $t=3.3$ μm, $s=0$ μm, and d ranging between 40 and 90 μm. For the analytical calculations we have used an effective radius of $R^*=4.7$ μm and $M_0=1.2\times 10^6$ A/m.

B. Dependence of the horizontal separation of the wires on the magnetostatic field

It is interesting to analyze the behavior of the magnetostatic plateaux as the interaxis distance, d, is kept fixed and the horizontal separation, s, is varied. Our results are combined in Fig. 5. Experimental data for the magnetostatic field of the system are depicted by gray dots and the solid line represents the analytical calculation. We observe a strong decrease in the magnetic field as the horizontal separation between wires is increased. Good agreement is obtained between the measured data and analytical calculations.

C. Interacting force

A couple of isolated magnetic microwires can attract or repel each other upon approach depending on their relative orientations. For the microwires investigated the interacting force, $\mathbf{F}=-\nabla E_{\mathrm{int}}$, has been estimated to be of the other of 0.017 (for $d=90$ μm and $s=0$ μm) to 0.0685 dyne (for $d=45$ μm and $s=0$ μm) for the case of interaxial separation in the range of parameters considered. For the horizontal separation, the interacting force is in the range between 1.63×10^{-5} (for $d=45$ μm and $s=3.5$ mm) to 0.0685 (for $d=45$ μm and $s=0$ mm) dyne.

V. DISCUSSION AND CONCLUSION

In summary, we have investigated the magnetostatic interaction between magnetic microwires. Using a continuous model we have obtained a simple expression to model the magnetostatic interaction in these particles. From our calculations and measurements we can conclude that the magnetostatic plateaux strongly depend on the geometry of the system. A couple of isolated magnetic microwires can attract or repel each other upon approach depending on their relative orientations. Based on microwires investigated the interacting force, $\mathbf{F}=-\nabla E_{\text{int}}$, has been estimated to be of the other of 1.63×10^{-5} (for $d=45$ μm and $s=3.5$ mm) to 0.0685 dyne (for $d=45$ μm and $s=0$ μm) in the range of parameters considered. The performance of such experiment could provide the basis for testing the different theoretical models. Our results provide guidelines for the production of microstructures with tailored magnetic properties.

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